INTERPOLACIÓN PARA PUNTOS EQUIESPACIADOS

Sea el conjunto de puntos $\{x_i, f(x_i)\}$, $i = 0, n$ tal que $x_i = x_0 + ih$, se definen las siguientes diferencias:

Diferencias Directas

$$\text{Orden 1}: \quad \Delta f(x) = f(x+h) - f(x); \qquad \text{Orden } n: \quad \Delta^n f(x) = \Delta^{n-1} f(x+h) - \Delta^{n-1} f(x)$$

Se verifican las propiedades

$$\Delta^n f(x) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} f(x_{n-j}); \qquad f[x_n, ..., x_0] = \frac{\Delta^n f(x_0)}{n!\, h^n}$$

Diferencias Inversas

$$\text{Orden 1}: \quad \nabla f(x) = f(x) - f(x-h); \qquad \text{Orden } n: \quad \nabla^n f(x) = \nabla^{n-1} f(x) - \nabla^{n-1} f(x-h)$$

Se verifica la propiedad

$$f[x_n, ..., x_0] = \frac{\nabla^n f(x_0)}{n!\, h^n}$$

Diferencias Centrales

$$\text{Orden 1}: \quad Df(x) = f(x+h/2) - f(x-h/2); \qquad \text{Orden } n: \quad D^n f(x) = D^{n-1} f(x+h/2) - D^{n-1} f(x-h/2)$$

Se verifican las propiedades

$$\text{si } n \text{ par}, \quad i = \frac{n}{2}, \quad f[x_n, ..., x_0] = \frac{D^n f(x_i)}{n!\, h^n}; \qquad \text{si } n \text{ impar}, \quad i = \frac{n-1}{2}, \quad f[x_n, ..., x_0] = \frac{D^n f(x_i + h/2)}{n!\, h^n}$$

FÓRMULAS DE INTERPOLACIÓN DE NEWTON Y GAUSS

Fórmula Directa de Newton

$$P_n(x_0 + \alpha h) = f(x_0) + \alpha \Delta f(x_0) + \frac{\alpha(\alpha-1)}{2} \Delta^2 f(x_0) + ... + \frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!} \Delta^n f(x_0)$$

$$R_n(x_0 + \alpha h) = \frac{\alpha(\alpha-1)...(\alpha-n)}{(n+1)!} \Delta^{n+1} f(x_0)$$

Fórmula Inversa de Newton

$$P_n(x_n + \alpha h) = f(x_n) + \alpha \nabla f(x_n) + \frac{\alpha(\alpha+1)}{2} \nabla^2 f(x_n) + ... + \frac{\alpha(\alpha+1)...(\alpha+n-1)}{n!} \nabla^n f(x_n)$$

$$R_n(x_n + \alpha h) = \frac{\alpha(\alpha+1)...(\alpha+n)}{(n+1)!} \nabla^{n+1} f(x_n)$$

Fórmula Directa de Gauss

$$\begin{aligned} P_n(x_m + \alpha h) = {} & f(x_m) + \alpha D f(x_m + h/2) + \frac{\alpha(\alpha-1)}{2} D^2 f(x_m) \\ & + \frac{\alpha(\alpha-1)(\alpha+1)}{3!} D^3 f(x_m + h/2) + \frac{\alpha(\alpha-1)(\alpha+1)(\alpha-2)}{4!} D^4 f(x_m) + ... \end{aligned}$$

Fórmula Inversa de Gauss

$$\begin{aligned} P_n(x_m + \alpha h) = {} & f(x_m) + \alpha D f(x_m - h/2) + \frac{\alpha(\alpha+1)}{2} D^2 f(x_m) \\ & + \frac{\alpha(\alpha+1)(\alpha-1)}{3!} D^3 f(x_m - h/2) + \frac{\alpha(\alpha+1)(\alpha-1)(\alpha+2)}{4!} D^4 f(x_m) + ... \end{aligned}$$