REGRESIÓN LINEAL

Modelo lineal: $\mathbf{E[Y|X=x]} = \alpha + \beta\mathbf{x}, \quad \mathbf{Y_i} = \alpha + \beta\mathbf{x_i} + \mathbf{E_i}$

Distribución de los estimadores

$\hat{\beta}$

$$\hat{\beta} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2}, \qquad E[\hat{\beta}] = \beta, \quad Var[\hat{\beta}] = \frac{\sigma^2}{nS_x^2}$$

Si n es grande (o las y_i son normales) y σ conocido

$$\frac{\hat{\beta} - \beta}{\frac{\sigma}{S_x\sqrt{n}}} \equiv N(0,1)$$

Si n es pequeño (σ desconocida)

$$\frac{\hat{\beta} - \beta}{\frac{S}{S_x\sqrt{n}}} \equiv t_{n-2}$$

$\hat{\alpha}$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}, \qquad E[\hat{\alpha}] = \alpha, \quad Var[\hat{\alpha}] = \frac{\sigma^2}{n}\left(1 + \frac{\bar{x}^2}{S_x^2}\right)$$

Si n es grande (o las y_i son normales) y σ conocido

$$\frac{\hat{\alpha} - \alpha}{\sqrt{\frac{\sigma^2}{n}\left(1 + \frac{\bar{x}^2}{S_x^2}\right)}} \equiv N(0,1)$$

Si n es pequeño (σ desconocida)

$$\frac{\hat{\alpha} - \alpha}{\sqrt{\frac{S^2}{n}\left(1 + \frac{\bar{x}^2}{S_x^2}\right)}} \equiv t_{n-2}$$

$\hat{\sigma}^2$

$$S^2 = \frac{1}{n-2}\sum\left(y_i - (\hat{\alpha} + \hat{\beta}x_i)\right)^2, \qquad E[S^2] = \sigma^2$$

$$\frac{(n-2)S^2}{\sigma^2} \equiv \chi^2_{n-2}$$