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**LINEAR ALGEBRA II****Exercises Unit IV. Chapter 1.****Conics.**(Academic Year 2022–2023)

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1.— Write the associated matrix to each one of the following conics:

i)  $x^2 - 4xy + 2y^2 - 4x + 6y + 4 = 0$ , ii)  $6xy - y^2 + 2x = 0$ , iii)  $(x + y)^2 - 5 = 0$ .

2.— Given the conic with equation  $x^2 - 4xy + 2y^2 - 4x + 6y + 4 = 0$

(a) find the tangent line to the conic at the point  $(2, 1)$ .

(b) find the exterior tangents to the conic through the point  $(0, -3)$ .

3.— Given the conic with equation  $x^2 + 4xy + y^2 - 4x - 6y = 0$ , find its center, asymptotes, axes, and vertices.

4.— Given the conic with equation  $x^2 + 4xy + y^2 - 4x - 6y = 0$  classify it and find its reduced equation and the corresponding change-of-reference equations.

5.— Given the conic with equation  $x^2 + 4xy + y^2 - 4x - 6y = 0$  find its foci, directrices and the eccentricity.

6.— Find the equation of a conic which passes through the points  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$ ,  $(1, 1)$ ,  $(2, 3)$ .

7.— Find the equation of a conic which passes through the points  $(1, 0)$ ,  $(2, 0)$ ,  $(0, 1)$  and is tangent to the line  $x + y - 4 = 0$  at point  $(2, 2)$ .

8.— Find the equation of a conic which is tangent to the line  $x + y - 1 = 0$  at the point  $(1, 0)$ , tangent to the line  $x - y + 3 = 0$  at the point  $(0, 3)$  and passes through the point  $(2, 1)$ .

9.— Find the equation of a conic which has as asymptotes the lines  $x + 2y - 1 = 0$ ,  $x - y = 0$  and passes through the point  $(2, -1)$ .

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**Solutions**

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1. i)  $\begin{pmatrix} 1 & -2 & -2 \\ -2 & 2 & 3 \\ -2 & 3 & 4 \end{pmatrix}$ . ii)  $\begin{pmatrix} 0 & 3 & 1 \\ 3 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ . iii)  $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -5 \end{pmatrix}$ .

2. (a)  $2xy - 3 = 0$ . (b)  $2xy - 3 = 0$ ,  $6xy - 3 = 0$ .

3. Center:  $(4/3, 1/3)$ .

Asymptotes:  $\sqrt{3}x + (2\sqrt{3} - 3)y + (1 - 2\sqrt{3}) = 0$ ,  $\sqrt{3}x + (2\sqrt{3} + 3)y - (1 + 2\sqrt{3}) = 0$ .

Axes:  $3x + 3y - 5 = 0$ ,  $xy - 1 = 0$ .

Vertices:  $((8 - \sqrt{22})/6, (2 - \sqrt{22})/6)$ ,  $((8 + \sqrt{22})/6, (2 + \sqrt{22})/6)$

4. Hyperbola. Reduced equation:  $\frac{x''^2}{11/9} - \frac{y''^2}{11/3} = 1$ .

Reference change:  $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 4/3 \\ 1/3 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ .

5. Foci:  $((4 + \sqrt{22})/3), (1 + \sqrt{22})/3$  and  $((4 - \sqrt{22})/3), (1 - \sqrt{22})/3$ .

Directrices:  $\sqrt{22}x + \sqrt{22}y - \frac{1}{3}(11 + 5\sqrt{22}) = 0$  y  $\sqrt{22}x + \sqrt{22}y + \frac{1}{3}(11 - 5\sqrt{22}) = 0$ .

Eccentricity: 2.

6.  $3x^2 - y^2 - 3x + y = 0$ .

7.  $2x^2 + xy + 2y^2 - 6x - 6y + 4 = 0$ .

8.  $7x^2 + 6xy + 3y^2 - 22x - 14y + 15 = 0$ .

9.  $x^2 + xy - 2y^2 - x + y + 3 = 0$ .