ÁLGEBRA LINEAL II — Exercises Unit II. Chapter 3

Orthogonal transformations. (Academic year 2022-2023)

1.– On the space $\mathbb{R}^2$ with the usual scalar product, find out which of the following linear mappings are orthogonal transformations:

(a) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad f(x, y) = (x + 2y, y)$

(b) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad f(x, y) = (-y, x)$

(c) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad f(x, y) = (\frac{3}{5}x - \frac{4}{5}y, \frac{4}{5}x + \frac{3}{5}y)$

(d) $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad f(x, y) = (x + y, x - y)$.

2.– Decide which of the following bases of $\mathbb{R}^3$ have the same orientation as the canonical basis.

(a) $B = \{(1, 0, 0),(0, 0, 1),(0, 1, 0)\}$.

(b) $B = \{(1, 1, 0),(1, 0, 2),(1, 1, 1)\}$.

(c) $B = \{(2, 3, 1),(1, -1, 0),(2, 0, 0)\}$.

3.– On $\mathbb{R}^2$ check which of the following pairs of bases have the same orientation:

(a) $B = \{(1, 2),(0, 1)\}$ and $B' = \{(2, 3),(1, 1)\}$.

(b) $B = \{(1, 1),(3, 0)\}$ and $B' = \{(-1, -1),(-3, 0)\}$.

(c) $B = \{(2, 0),(1, 3)\}$ and $B' = \{(2, 3),(1, 5)\}$.

4.– Decide whether each of the following orthogonal transformations of $\mathbb{R}^2$ with the usual scalar product is direct or inverse.

(a) The transformation whose associated matrix relative to the canonical basis is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(b) The transformation whose associated matrix relative to the canonical basis is $\begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$.

(c) The transformation whose associated matrix relative to the canonical basis is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

5.– On $\mathbb{R}^2$, with the usual scalar product and taking as positive the orientation given by the canonical basis, give the matrix associated to a rotation of 120^o.

6.– On $\mathbb{R}^2$, with the usual scalar product, give the matrix associated to the symmetry with respect to the line $\mathcal{L}\{(1, 2)\}$.

7.– On $\mathbb{R}^3$, with the usual scalar product and taking as positive the orientation given by the canonical basis, give the matrix associated to a rotation of 30^o around the semi-axis $\mathcal{L}\{(0, 1, 0)\}$.

8.– On $\mathbb{R}^3$, with the usual scalar product, give the matrix associated to the symmetry with respect to the plane $\mathcal{L}\{(0,1,2),(1,0,0)\}$.

9.– On $\mathbb{R}^2$, with the usual scalar product and taking as positive the orientation given by the canonical basis, classify the orthogonal transformations in exercise (4).

Soluciones.

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1. (a) No. (b) Yes. (c) Yes. (d) No.

2. (a) No. (b) No. (c) Yes.

3. (a) No. (b) Yes. (c) Yes.

4. (a) Direct. (b) Direct. (c) Inverse.

5. $\begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$.

6. $\begin{pmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{pmatrix}$.

7. $\begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix}$.

8. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -3/5 & 4/5 \\ 0 & 4/5 & 3/5 \end{pmatrix}$.

9. (a) It is a rotation of 90 degrees.

(b) It is a rotation of angle $+arcos(3/5)$.

(c) It is a symmetry with respect to the line $\mathcal{L}\{(1,1)\}$.